Lessons 026 - 028

Wednesday, November 15

If the same process is used to construct two confidence intervals, which of the following statements is false?

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If n increases, everything else equal, then the second interval will be shorter.

0%

If α increases, everything else equal, then the second interval will be shorter.

0%

If the confidence level increases, everything else equal, then the second interval will be longer.

0%

If the standard error decreases, everything else equal, then the second interval will be shorter.

0%

If the same process is used to construct two confidence intervals, which of the following statements is false?

0

In most settings, if n increases, everything else equal, then the second interval will be shorter.

0%

If α increases, everything else equal, then the second interval will be shorter.

0%

If the confidence level increases, everything else equal, then the second interval will be longer.

0%

If the standard error decreases, everything else equal, then the second interval will be shorter.

0%

All of the statements are true.

0%

Introductory statistics is weird ...

Confidence Intervals with Unknown Variance

- If we assume that σ^2 is known then we can standardize for normal confidence intervals

$$\frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

- If σ^2 is unknown, this will not work.

- If we replace σ^2 with s^2, the sampling distribution will no longer be normal.

Approximation with CLT (Large Sample)

- If n is sufficiently large, then we can approximate this with the CLT.
- Recall that for large n, $\overline{X} \dot{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$ and as such

$$\frac{\overline{X} - \mu}{s/\sqrt{n}} \dot{\sim} N(0,1)$$

- This can be used as approximately correct for large samples.

Approximation with CLT (Small Sample)

- If n is small then this approximation is invalid.

- If the population is normally distributed, n is small, and σ^2 is unknown, then we can form confidence intervals using a new distribution

$$\frac{\overline{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}$$

The t-Distribution

- The t-distribution is a visually similar distribution to the normal, but with more density in the tails.

- The parameter, denoted ν, is referred to as the **degrees of freedom**.

- As we take $\nu \to \infty$ then we have that $t_\nu \to N(0,1)$

- This approximation is fairly accurate for $\nu \approx 30$.

0.0
0.1
0.2
0.3
0.4
0.5
Density
−3
−2
−1
0
1
2
3
x
df = 2
df = 5
df = 10
df = 30
N(0,1)

Confidence Intervals with the t-Distribution

- Just as we have Z_α critical values for the standard normal, we have $t_{\alpha,\nu}$ critical values for the t distribution.
- Thus, an equivalent argument as before gives a confidence interval that takes on nearly the same form

$$\overline{X} \pm t_{\alpha/2,\nu}\frac{s}{\sqrt{n}}$$

Confidence Intervals for Proportions

- Recall that if $X \sim \text{Bin}(n, p)$, then if n is large enough we have that $X \mathrel{\dot\sim} N(np, np(1-p))$.
- Using this we find that $\hat{p} = \dfrac{X}{n} \mathrel{\dot\sim} N\left(p, \dfrac{p(1-p)}{n}\right)$.
- Thus, the arguments we have been making give an easy method for confidence intervals for proportions since

$$\frac{\hat{p} - p}{\sqrt{p(1-p)/n}} \mathrel{\dot\sim} N(0,1).$$

Suppose that a sample of size $n = 15$ is taken from a normal distribution, with σ^2 known. What is the relevant distribution for resulting confidence intervals?

Normal Distribution

0%

t-distribution with 15 degrees of freedom.

0%

t-distribution with 14 degrees of freedom.

0%

None of the above, or insufficient information.

0%

Suppose that a sample of size $n = 15$ is taken from a normal distribution, with σ^2 unknown. What is the relevant distribution for resulting confidence intervals?

Normal Distribution

0%

t-distribution with 15 degrees of freedom.

0%

t-distribution with 14 degrees of freedom.

0%

None of the above, or insufficient information.

0%

Suppose that a sample of size $n = 15$ is taken from a non-normal distribution, with σ^2 unknown. What is the relevant distribution for resulting confidence intervals?

Normal Distribution

0%

t-distribution with 15 degrees of freedom.

0%

t-distribution with 14 degrees of freedom.

0%

None of the above, or insufficient information.

0%

Suppose that a sample of size $n = 150$ is taken from a non-normal distribution, with σ^2 unknown. What is the relevant distribution for resulting confidence intervals?

Normal Distribution

0%

t-distribution with 150 degrees of freedom.

0%

t-distribution with 149 degrees of freedom.

0%

None of the above, or insufficient information.

0%

General Procedure for Confidence Intervals

- If ever we have a quantity, $T(\widehat{\theta}, \theta)$, which has a distribution that is functionally independent of θ, we can use this to form a confidence interval.
- For instance:

$$T(\widehat{\mu}, \mu) = \frac{\overline{X} - \mu}{s/\sqrt{n}} \sim t_{n-1}.$$

$$T(\widehat{p}, p) = \frac{\widehat{p} - p}{\sqrt{p(1-p)/n}} \dot{\sim} N(0,1)$$

- How does this work?

Example of General Procedure and CIs for σ^2.